

MUTI-EXTENT 5-PART SURPRISE MINOR

After ringing a peal of the standard 41 Surprise Minor with a change of backwork every lead in December 2011, it was suggested by a couple of the band that the next challenge should be an all-the-work version with the same feature; the composition we had just rung was based on 3-part callings, so the two observation bells in each part (5 and 6) didn't experience the same complexity as the other 3 working bells. At the time I thought this would be difficult to accomplish, existing all-the-work compositions containing all 41 methods required a minimum of 7200 changes and were primarily based on full-course splicing, a format which would be difficult, if not impossible, to rearrange in such a way as to include a change of backwork every lead.

One possible solution that came to mind, however, was to produce a 5-part composition which, in the same way as 7-part compositions of Major, would automatically give all-the-work, as each working bell would ring the same part from each of the 5 different starting positions. Although peals of Minor do not need to be made up of "true" 720's, they do need to comprise each of the possible 720 rows an equal number of times (e.g. each possible row needs to occur 7 times in a 5040) which makes the job of producing a composition much harder than for Major where you have 40320 rows to pick from. To my knowledge, no one had really looked into multi-extent 5-part compositions of Minor before (other than ones based on multiples of true 720s) and I soon realised why; many of the existing splicing techniques I was familiar with would be of no help in a 5-part format, a different approach was going to be required which virtually required starting from scratch.

My starting point was to understand how existing 5-part 720s worked. The simplest example is a 720 of spliced Cambridge and Primrose as follows (this is an arrangement by A. Relfe):

Part 1	Part 2	Part 3	Part 4	Part 5
<u>23456</u> Cm	<u>42635</u> Cm	<u>64523</u> Cm	<u>56342</u> Cm	<u>35264</u> Cm
- <u>35642</u> Cm	- <u>23564</u> Cm	- <u>42356</u> Cm	- <u>64235</u> Cm	- <u>56423</u> Cm
42563 Pr	64352 Pr	56234 Pr	35426 Pr	23645 Pr
26435 Pr	45623 Pr	63542 Pr	52364 Pr	34256 Pr
63254 Cm	52436 Cm	34625 Cm	26543 Cm	45362 Cm
54326 Cm	36245 Cm	25463 Cm	43652 Cm	62534 Cm
- <u>42635</u>	- <u>64523</u>	- <u>56342</u>	- <u>35264</u>	- <u>23456</u>

All true 720s of Surprise Minor consist of all 5 of the leads in each of the 6 courses starting from 23456, 42356, 34256, 54326, 53246 and 52436. The 5-part composition above works because each part contains 1 lead from the plain course 23456 (a different lead in each part) plus all 5 leads from one of the other 5 courses (a different course in each part, course 53246 is rung as its reverse 35426), so by the time you reach the end of the 720 you have rung all 5 leads of each of the 6 courses. For the courses other than the plain course, some of the leads are rung as Cambridge and some as Primrose, this particular arrangement of the leads enables you to ring all 5 leads of the course but at the end return back to the plain course at a lead different to the one you started from.

By extrapolation we can determine that a 5-part 1440 can be constructed by having two occurrences of each lead in each part, i.e. each part must have 2 leads from the plain course plus a further 10 leads which between them give you 2 occurrences of each lead from the other 5 courses. A simple example of this would be as follows, where the starting point is an extent each of Cambridge and York, these are merged together into a 5-part 1440 where each part contains 1 lead from the plain course of each method, and all 5 leads of 1 of the other courses of each method:

Part 1		Part 2		Part 3		Part 4		Part 5	
A1	<u>23456</u> Cm	A1	<u>64523</u> Cm	A1	<u>35264</u> Cm	A1	<u>42635</u> Cm	A1	<u>56342</u> Cm
A2	<u>56342</u> Yo	A2	<u>23456</u> Yo	A2	<u>64523</u> Yo	A2	<u>35264</u> Yo	A2	<u>42635</u> Yo
B1	- <u>56423</u> Cm	B1	- <u>23564</u> Cm	B1	- <u>64235</u> Cm	B1	- <u>35642</u> Cm	B1	- <u>42356</u> Cm
C1	<u>23645</u> Cm	C1	<u>64352</u> Cm	C1	<u>35426</u> Cm	C1	<u>42563</u> Cm	C1	<u>56234</u> Cm
D1	<u>45362</u> Cm	D1	<u>52436</u> Cm	D1	<u>26543</u> Cm	D1	<u>63254</u> Cm	D1	<u>34625</u> Cm
E1	<u>62534</u> Cm	E1	<u>36245</u> Cm	E1	<u>43652</u> Cm	E1	<u>54326</u> Cm	E1	<u>25463</u> Cm
F1	- <u>34256</u> Cm	F1	- <u>45623</u> Cm	F1	- <u>52364</u> Cm	F1	- <u>26435</u> Cm	F1	- <u>63542</u> Cm
F2	- <u>45623</u> Yo	F2	- <u>52364</u> Yo	F2	- <u>26435</u> Yo	F2	- <u>63542</u> Yo	F2	- <u>34256</u> Yo
D2	<u>52436</u> Yo	D2	<u>26543</u> Yo	D2	<u>63254</u> Yo	D2	<u>34625</u> Yo	D2	<u>45362</u> Yo
B2	<u>23564</u> Yo	B2	<u>64235</u> Yo	B2	<u>35642</u> Yo	B2	<u>42356</u> Yo	B2	<u>56423</u> Yo
E2	<u>36245</u> Yo	E2	<u>43652</u> Yo	E2	<u>54326</u> Yo	E2	<u>25463</u> Yo	E2	<u>62534</u> Yo
C2	<u>64352</u> Yo	C2	<u>35426</u> Yo	C2	<u>42563</u> Yo	C2	<u>56234</u> Yo	C2	<u>23645</u> Yo
	- <u>64523</u>		- <u>35264</u>		- <u>42635</u>		- <u>56342</u>		- <u>23456</u>

By the time you reach the end of the 1440 you have rung all 5 leads of each of the 6 courses of Cambridge, and all 5 leads of each of the 6 courses of York. In this example each of the leads in the 5-lead blocks of Cambridge and York in each part come from the same course, but that need not be the case. If we give a unique label to each of the 12 leads in each part (A1, A2, B1, B2, etc. in the example above), and treat all 5 occurrences of each label as a group (e.g. group E2 contains leads 36245, 43652, 54326, 25463 and 62534 of York) then we can construct alternative callings which contain any 1 lead from each group in each part; as long as 1 lead from each of the 12 groups is included, and the calling finishes by returning to a different lead of the plain course to the one you started from, then a true 1440 will be achieved, because the remaining 4 members of each group will automatically occur in the remaining 4 parts of the composition. An example of this would be as follows:

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23456 Cm   A1 (part 1)
56342 Yo   A2 (part 1)
- 56423 Cm B1 (part 1)
23645 Yo   C2 (part 5)
34256 Cm   F1 (part 1)
- 45623 Yo F2 (part 1)
- 45236 Yo E2 (part 3 - lead 54326 reversed)
- 45362 Yo D2 (part 5)
56423 Yo   B2 (part 5)
- 56234 Cm C1 (part 5)
34625 Cm   D1 (part 5)
25463 Cm   E1 (part 5)
- 56342
Repeat 4x

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The ability to pick and choose which of the 5 leads in a group to include in each part means that existing known true 5-part blocks which cannot be constructed as a 5-part 720 can be combined into a 5-part 1440. For example, there are 2 known Cambridge-over extents which have a 5-part structure but cannot be arranged into 5-part callings. These are:

A1 <u>23456</u> Cm	A2 <u>23456</u> Bo
B1 - <u>35642</u> Nf	B2 - <u>56423</u> Du
F1 <u>26435</u> Pr	E2 <u>62534</u> Hu
D1 <u>63254</u> Nf	F2 <u>34256</u> Bk
C1 <u>42563</u> Ip	D2 <u>45362</u> Bo
E1 <u>54326</u> Yo	C2 <u>23645</u> Du
- <u>54263</u>	- <u>23456</u>

In the first block, the leads from groups B1, F1, D1, C1 and E1 cannot be arranged in such a way as to bring you back to a lead of the plain course. In the second block, the only lead of the plain course which the leads from groups B2, E2, F2, D2 and C2 can be arranged to bring you back to is the lead 23456 which is the same one you started from so you are unable to reach the remaining 4 parts of the calling. But the two callings can be combined into a 1440 as follows:

A1 <u>23456</u> Cm	
A2 <u>56342</u> Bo	
B2 - <u>42356</u> Du	
C1 - <u>42563</u> Ip	
E1 <u>54326</u> Yo	
B1 - <u>54263</u> Ip	(lead 23564 reversed)
F1 <u>25346</u> Cm	(lead 45623 reversed)
D1 <u>46532</u> Ip	(lead 52436 reversed)
E2 - <u>25463</u> Hu	
F2 <u>63542</u> Bk	
D2 <u>34625</u> Bo	
C2 <u>56234</u> Du	
- <u>56342</u>	
Repeat 4x	

All this helped me to understand how a multi-extent 5-part block would need to be constructed. For example, each part of a composition of 2160 changes would require 3 leads from the plain course plus a further 15 leads which between them give you 3 occurrences of each lead from the other 5 courses; a composition of 2880 changes would require 4 occurrences of everything, etc, etc.

In the example 1440s above the building blocks had all been true 720s in their own right which had been merged together. These can lead to some quite interesting and varied callings but limit you in respect of the number of methods you can include; the advantage of multi-extent compositions is that your building blocks do not need to be true 720s, the only requirement is that each of the 720 different rows occurs the same number of times. So in theory you could have two separate 5-part blocks of 720 changes in each of which some of the 720 rows occurred just once, some occurred twice, and some didn't occur at all, but all the rows which occurred twice in the first block were the same rows which didn't occur at all in the second block, and all the rows which occurred twice in the second block were the same rows which didn't occur at all in the first block – when rung together as a 1440 the 2 blocks would give you 2 occurrences of every row. How was I going to find an actual example of this theory?

I decided that the best approach would be to construct my own 6-lead blocks, each of which would consist of 1 lead of the plain course and then all 5 leads of one of the other courses (I chose the course 54326), and then try to find blocks which complemented each other in terms of the duplicate and missing rows. All the methods in each block would have the same backwork, and the 5 leads chosen for the 54326 course needed to include all 10 of the lead-heads and lead-ends required for that course, i.e. 54326, 53462, 42563, 45236, 26435, 24653, 63254, 62345, 35642 and 36524; this would mean that there would be no duplicated or missing rows in the treble's 1-2 and 3-4 sections, the only "issues" would be in the middle six rows of each lead in the treble's 5-6 sections. There was no requirement that the leads chosen for the 54326 course should be able to be arranged in such a way as to bring you back to the plain course as the earlier example 1440s illustrate. An example of a 6-lead block which I derived was as follows:

Plain Course Lead	Bamborough	
Lead-head 26435	Wearmouth	Lead-end 36524
Lead-head 35642	Bamborough	Lead-end 62345
Lead-head 63254	Bamborough	Lead-end 24653
Lead-head 42563	Norwich	Lead-end 45236
Lead-head 54326	Norwich	Lead-end 53462

I needed to invent a method of determining quickly which rows in the 5-6 sections of the block were repeated and which were missing, so I came up with a way of categorising the rows in the 5-6 sections. To do this I used the 5-part 720 of Cambridge and Primrose as my "master" block and assigned unique group codes to each of the negative rows in the twelve 5-6 sections of each part (codes A-L for the rows with the treble in 6ths, and codes 01-12 for the rows with the treble in 5ths). There was no need to analyse the positive rows with the treble in 5ths because the construction of the blocks meant these would always be unique. There was also no need to analyse the positive rows with the treble in 6ths separately, as these are always paired uniquely with negative rows with the treble in 5ths via an "X" in the place notation regardless of the method being rung. As can be seen from the following table, the rows in parts 2 to 5 of the composition were assigned the same codes as their counterparts in part 1 of the composition so each code consisted of a group of 5 different rows.

	23456 Cm		42635 Cm		64523 Cm		56342 Cm		35264 Cm
A	624513 +	A	546312 +	A	365214 +	A	253416 +	A	432615 +
01	265431 -	01	453621 -	01	632541 -	01	524361 -	01	346251 -
	256413 -		435612 -		623514 -		542316 -		364215 -
02	<u>524631</u> +	02	<u>346521</u> +	02	<u>265341</u> +	02	<u>453261</u> +	02	<u>632451</u> +
B	256431 +	B	435621 +	B	623541 +	B	542361 +	B	364251 +
	524613 -		346512 -		265314 -		453216 -		632415 -
	542631 -		364521 -		256341 -		435261 -		623451 -
	456213 +		635412 +		523614 +		342516 +		264315 +
	35642 Cm		23564 Cm		42356 Cm		64235 Cm		56423 Cm
C	236415 +	C	425613 +	C	643512 +	C	562314 +	C	354216 +
03	324651 -	03	246531 -	03	465321 -	03	653241 -	03	532461 -
	342615 -		264513 -		456312 -		635214 -		523416 -
04	<u>436251</u> +	04	<u>625431</u> +	04	<u>543621</u> +	04	<u>362541</u> +	04	<u>254361</u> +
D	342651 +	D	264531 +	D	456321 +	D	635241 +	D	523461 +
	436215 -		625413 -		543612 -		362514 -		254316 -
	463251 -		652431 -		534621 -		326541 -		245361 -
	642315 +		564213 +		356412 +		235614 +		423516 +
	42563 Pr		64352 Pr		56234 Pr		35426 Pr		23645 Pr
E	345612 +	E	263514 +	E	452316 +	E	634215 +	E	526413 +
05	436521 -	05	625341 -	05	543261 -	05	362451 -	05	254631 -
	463512 -		652314 -		534216 -		326415 -		245613 -
06	<u>645321</u> +	06	<u>563241</u> +	06	<u>352461</u> +	06	<u>234651</u> +	06	<u>426531</u> +
F	463521 +	F	652341 +	F	534261 +	F	326451 +	F	245631 +
	645312 -		563214 -		352416 -		234615 -		426513 -
	654321 -		536241 -		325461 -		243651 -		462531 -
	563412 +		352614 +		234516 +		426315 +		645213 +
	26435 Pr		45623 Pr		63542 Pr		52364 Pr		34256 Pr
G	524316 +	G	346215 +	G	265413 +	G	453612 +	G	632514 +
07	253461 -	07	432651 -	07	624531 -	07	546321 -	07	365241 -
	235416 -		423615 -		642513 -		564312 -		356214 -
08	<u>324561</u> +	08	<u>246351</u> +	08	<u>465231</u> +	08	<u>653421</u> +	08	<u>532641</u> +
H	235461 +	H	246351 +	H	465231 +	H	653421 +	H	532641 +
	324516 -		246315 -		465213 -		653412 -		532614 -
	342561 -		264351 -		456231 -		635421 -		523641 -
	435216 +		623415 +		542613 +		364512 +		256314 +
	63254 Cm		52436 Cm		34625 Cm		26543 Cm		45362 Cm
I	462513 +	I	654312 +	I	536214 +	I	325416 +	I	243615 +
09	645231 -	09	563421 -	09	352641 -	09	234561 -	09	426351 -
	654213 -		536412 -		325614 -		243516 -		462315 -
10	<u>562431</u> +	10	<u>354621</u> +	10	<u>236541</u> +	10	<u>425361</u> +	10	<u>643251</u> +
J	654231 +	J	536421 +	J	325641 +	J	243561 +	J	462351 +
	562413 -		354612 -		236514 -		425316 -		643215 -
	526431 -		345621 -		263541 -		452361 -		634251 -
	254613 +		436512 +		625314 +		543216 +		362415 +
	54326 Cm		36245 Cm		25463 Cm		43652 Cm		62534 Cm
K	653214 +	K	532416 +	K	324615 +	K	246513 +	K	465312 +
11	562341 -	11	354261 -	11	236451 -	11	425631 -	11	643521 -
	526314 -		345216 -		263415 -		452613 -		634512 -
12	<u>253641</u> +	12	<u>432561</u> +	12	<u>624351</u> +	12	<u>546231</u> +	12	<u>365421</u> +
L	526341 +	L	345261 +	L	263451 +	L	452631 +	L	634521 +
	253614 -		432516 -		624315 -		546213 -		365412 -
	235641 -		423561 -		642351 -		564231 -		356421 -
	326514 +		245316 +		463215 +		652413 +		534612 +

The next step was to code my own 6-lead block of Bamborough, Wearmouth and Norwich in the same way, annotating each 5-6 section negative row with its corresponding group code (see the following page). It does not matter which of the 5 parts of the master Cambridge/Primrose grid the row belongs to, it is just the group that is important; this is because, if my own block was rung 5 times with each part starting from a different lead of the plain course, the other 4 members of each group would occur automatically in the subsequent 4 parts of the composition. Consider for example the Bamborough row 623451 which belongs to group B; in the part starting from 42635 this row will be 542631, in the part starting from 64523 this row will be 364521, in the part starting from 56342 this row will be 256341, and in the part starting from 35264 this row will be 435261; so we get the complete set of group B rows albeit in a different order to the master Cambridge/Primrose grid.

Looking at the groups included in my 6-lead block, there was 1 occurrence each of groups A, B, C, L, 01, 02, 03 and 12, two occurrences each of groups F, G, H, I, 06, 07, 08 and 09, and no occurrences of groups D, E, J, K, 04, 05, 10 and 11. So to construct a true 1440 I now needed to find a complementary 6-lead block which also had 1 occurrence each of groups A, B, C, L, 01, 02, 03 and 12, but had no occurrences of groups F, G, H, I, 06, 07, 08 and 09, and had 2 occurrences each of groups D, E, J, K, 04, 05, 10 and 11. Well it turns out there was one as follows:

Plain Course Lead	Bourne	
Lead-head 54326	Cambridge	Lead-end 24653
Lead-head 26435	Durham	Lead-end 62345
Lead-head 63254	Durham	Lead-end 36524
Lead-head 35642	Cambridge	Lead-end 45236
Lead-head 42563	Ipswich	Lead-end 53462

Neither of the 6-lead blocks can be arranged as a 5-part calling individually, but together they can be combined into a true 5-part 1440, e.g.

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A1  23456 Bo
A2  64523 Bm
B1  - 35642 Cm
C1  42563 Ip
E1  54326 Cm
F1  26435 Du
D1  63254 Du
F2  - 63542 Wm
B2  42356 Bm
C2  - 63425 No
E2  - 46325 No
D2  - 34625 Bm
    - 56342
Repeat 4x

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On the face of it, it's not particularly obvious why these 2 blocks should complement each other. But if you do want to dig a little deeper, you will find that the leads of Bourne, Durham, Wearmouth and Norwich between them contain 1 occurrence of each group code, and likewise the leads of Bamborough, Cambridge and Ipswich; it is no coincidence that the first group all have the place notation 34 between the negative rows in the 5-6 section, and the second group all have the place notation 14.

06 F I 09	23456 Bm
	324615 + 326451 + 234615 - <u>243651</u> - 426351 - 462315 - 643251 + 642315 +
	26435 Wm
	624513 + 642531 + 465213 - <u>645231</u> - 654321 - 564312 - 653421 + 635412 +
	35642 Bm
	536214 + 532641 + 356214 - <u>365241</u> - 632541 - 623514 - 265341 + 263514 +
02 B H 08	63254 Bm
	362415 + 364251 + 632415 - <u>623451</u> - 264351 - 246315 - 423651 + 426315 +
	42563 No
	245316 + 423561 - 243516 - <u>425361</u> + 452631 + 546213 - 456231 - 542613 +
G 03 06 C	54326 No
	453612 + 546321 - 456312 - <u>543621</u> + 534261 + 352416 - 532461 - 354216 +

A 04 11 B	23456 Bo
	624513 + 265431 - 625413 - <u>264531</u> + 624351 + 263415 - 623451 - 264315 +
	54326 Cm
	653214 + 562341 - 526314 - <u>253641</u> + 526341 + 253614 - 235641 - 326514 +
02 E D 10	26435 Du
	524316 + 542361 + 453216 - <u>543261</u> - 534621 - 354612 - 536421 + 563412 +
	63254 Du
05 K J 01	462513 + 426531 + 245613 - <u>425631</u> - 452361 - 542316 - 453261 + 435216 +
	35642 Cm
C 03 04 D	236415 + 324651 - 342615 - <u>436251</u> + 342651 + 436215 - 463251 - 642315 +
	42563 Ip
E 05 10 J	345612 + 436521 - 463512 - <u>645321</u> + 654231 + 562413 - 526431 - 254613 +

In order to determine what other sets of complementary blocks might exist, I first needed to build a comprehensive library of all the possible 6-lead blocks containing 1 lead from the plain course and all 5 leads from the course 54326. All the Carlisle-over and London-over blocks were built as their out-of-course equivalents, so instead they contained 1 lead from the course 45326 and all 5 leads from the course 32456, this meant that the parity of the rows in the 5-6 sections would match up with the Cambridge-over and Norwich-over blocks. It goes without saying that all the blocks contained 2 or more methods.

While building the blocks I realised that it was possible in specific circumstances to combine Cambridge-over and Norwich-over methods in a block, and likewise Carlisle-over and London-over methods, whilst still retaining truth in the 1-2 and 3-4 sections. If a block contained a Cambridge-over method for the lead of the plain course, then the methods rung in the 54326 course containing lead-heads 54326 and 26435 and lead-ends 36524 and 45236 also needed to be rung as Cambridge-over methods, but the remaining 3 leads of the 54326 course could all be rung as Norwich-over methods. An example of this would be:

Plain Course Lead	York	
Lead-head 54326	Ipswich	Lead-end 36524
Lead-head 26435	Ipswich	Lead-end 45236
Lead-head 63254	Bamborough	Lead-end 24653
Lead-head 35642	Allendale	Lead-end 53462
Lead-head 42563	Lightfoot	Lead-end 62345

If the block contained a Norwich-over method for the lead of the plain course then the reverse logic would apply.

If a block contained a Carlisle-over method for the lead of the course 45326, then the methods rung in the 32456 course containing lead-heads 25364 and 43625 and lead-ends 52634 and 34265 also needed to be rung as Carlisle-over methods, but the remaining 3 leads of the 32456 course could all be rung as London-over methods. An example of this would be:

Course 45326 Lead	Alnwick	
Lead-head 25364	Sandiacre	Lead-end 34265
Lead-head 43625	Alnwick	Lead-end 52634
Lead-end 64532	Cunecastre	Lead-end 46352
Lead-end 56243	Wells	Lead-end 23546
Lead-head 32456	Kelso	Lead-end 65423

If the block contained a London-over method for the lead of the course 45326 then the reverse logic would apply.

If you consider all the different lead-end groups (regardless of whether 2nds or 6ths is made at the lead-end) there are 26 different formats of the "54326" course, each of which has 5 rotations (i.e. starting from 54326, 42563, 26435, 63254 or 35642); any lead-end group can be chosen for the lead from the plain course. The 26 different formats of the "54326" course are illustrated below, using Norwich-over methods as examples as all the different lead-end groups are represented by these methods.

Update July 2014: It was pointed out to me by Simon Humphrey that there are in fact 28 different formats of the "54326" course, however these 2 additional course formats don't seem to add anything new.

No.	54326	42563	26435	63254	35642
1	Ad	Ad	Ad	Ad	Ad
2	Ad	Bm	Ad	Ad	Ne
3	Ad	Ad	Lf	No	Ad
4	Ne	Ad	Lf	Ne	Ad
5	Lf	Bm	Ad	Ad	Lf
6	Bm	Ad	Bm	No	Ad
7	No	No	Ad	Ad	Ne
8	Ne	Ad	Lf	Lf	Lf
9	Ad	Ne	Ne	Bm	Bm
10	Ad	Bm	Lf	Bm	Bm
11	Ad	Ne	Ne	No	Ne
12	Ad	Bm	Lf	No	Ne
13	Ad	Lf	No	Lf	No
14	Ad	Bm	No	No	No
15	Lf	Lf	Lf	Lf	Lf
16	Ne	Ne	Ne	Ne	Ne
17	Ne	Ne	Bm	Lf	Ne
18	Bm	Lf	Bm	Lf	Ne
19	Bm	Bm	Bm	Bm	Bm
20	Lf	Ne	Ne	No	Lf
21	Lf	Bm	Lf	No	Lf
22	Bm	Bm	Bm	No	Ne
23	Ne	Bm	No	Ne	No
24	No	No	Lf	Bm	Bm
25	No	No	Lf	No	Ne
26	No	No	No	No	No
27	No	Ad	Lf	Ne	Bm
28	No	Lf	Bm	Ad	Ne

This meant that the total number of different 6-lead blocks in my resulting library was in excess of 10,000, all of which then needed to be coded up to show which negative rows occurred in the 5-6 sections. Determining all the possible blocks obviously took quite a while, a simple lookup from a master table containing all possible leads of each method was used to determine the group codes contained in each block. Given the large number of blocks involved, trying to find complementary sets by hand obviously wasn't practical, so I uploaded the details of all my blocks into a database so that I could then write queries to find "true" complementary sets.

The first search was for 1440s, i.e. pairs of blocks which between them contained each of the twelve 5-6 section group codes twice. Many of these turned out to be two true 720s merged together so some work was needed to weed out the “new” sets. Some of the more interesting ones can be found on my website. One which I found quite interesting was the following 1440 because it contained 3 leads of each backwork (the codes beside each lead show the groups contained in the two 5-6 sections of each lead):

1440	23456	Yo	H	08	G	07	45326	Lo	D	04	K	11
	26435	Cm	G	07	H	08	64532	Nw	D	10	E	02
	35642	Ne	I	09	F	06	32456	Nw	J	01	K	05
	54326	Du	A	04	B	11	43625	Cu	L	12	B	02
	42563	Lf	C	03	I	09	56243	Nb	J	10	E	05
	63254	Lf	F	06	L	12	25364	Cu	A	01	C	03

These leads can be arranged into a 1440 as follows:

```

23456 Yo
s 32564 Nw
  45632 Cu
s 54326 Du
  42563 Lf
s 62354 Lo
s 34625 Lf
s 24563 Nb
  63452 Cu
s 36524 Cm
  24653 Ne
s 35624 Nw
s 42635
Repeat 4x

```

I then did a similar search for 2160s, this obviously took considerably longer and came up with a large number of results, too many to examine individually. Of course the ultimate goal was to get a 5-part containing all 41 methods, so I then started looking for combinations of the 1440s and 2160s found so far which together might allow all 41 methods to be included. Despite there being 42 leads in each part of a 5-part 5040, the maximum I could get out by combining the 1440s and 2160s (and also 720s) was disappointingly just 39 methods. However, assuming these leads could be arranged suitably in a 5-part format this would still exceed the existing maximum of 36 methods all-the-work in a normal peal length. Additionally, it appeared all 41 methods could be obtained in combinations of 2160s and 1440s provided you had 8 blocks in total giving a peal length of 5760 changes, this would significantly reduce the existing minimum length of 7200 changes required to ring all 41 all-the-work.

It would seem in theory that it might be possible to get all 41 methods into a normal peal length containing 7 blocks. However I could tell from the length of time taken for the 1440 and 2160 searches to run that running similar searches for 2880s, 3600s, 4320s or even 5040s would be impossible, so I needed to find some shortcuts. The overall format of a 7-block 5040 would need to include 1 London-over block and 2 blocks each of Norwich-over, Cambridge-over and Carlisle-over to allow all the methods to be included; mixed blocks (i.e. Cambridge and Norwich, or Carlisle and London) would also work as long as the total number of leads of each backwork came to the same number. I managed to perform a search for 2880s in a reasonable timescale using this backwork restriction together with the condition that they must be able to yield at least 23 methods; combined with a 2160 of 17 or 18 methods this might give a total of 41 – alas it did not, the maximum total was still stuck stubbornly on 39.

I thought of a further restriction on the search that I could make. For the London-over leads to include all 6 London-over methods, the only block format which could achieve this was one where the lead from course 45326 was rung as Cunecastre, and the 32456 course was made up of leads of London, Wells, Kelso, Lincoln and Coldstream; no other format for the London-over leads would work. So the final search I did worked as follows:

- (a) Determine the 6-lead London-over blocks which met the required criteria stated above and would yield all 6 London-over methods (there were 10 of these).
- (b) Combine these results with 4 blocks of Norwich-over and/or Cambridge-over methods which between them yielded all 24 Norwich-over and Cambridge-over methods and which, when combined with the (a) blocks did not include any one 5-6 section group more than 7 times.
- (c) Combine these results with pairs of Carlisle-over blocks which between them yielded all 11 Carlisle-over methods and which, when combined with the (a) and (b) blocks included every 5-6 section group exactly 7 times.

Unfortunately this search yielded no results. Having given up on the idea of getting all 41 methods into 5040 changes I tried modifying the search to allow for 1 missing method (i.e. potentially getting 40 methods) but this search was clearly going to take an age to complete so I gave up on it.

So in theory, compositions containing 39 methods in 5040 changes or 41 methods in 5760 changes were now possible, the next step was to starting building the compositions.

The starting point of all this had been to try to get a 41 all-the-work peal with a change of backwork every lead, and to achieve this some care would need to be taken in choosing suitable blocks. After a lot of analysis, the set of blocks which I chose for this in the end were as follows, two 2160-blocks and one 1440-block:

2160	23456	Du	I	08	F	07		45326	Ch	F	07	I	08		23456	Ne	K	11	D	04
	42563	Bv	L	09	D	10		52634	Nw	A	04	B	11		35642	Wm	K	05	J	01
	63254	Yo	E	05	A	01		46352	Sa	E	02	H	12		42563	Lf	C	03	I	09
	35642	Bv	H	12	B	11		65423	Ch	K	05	C	06		26435	Lf	H	08	G	07
	54326	Bv	A	04	G	03		34265	Sa	C	06	L	09		63254	Lf	F	06	L	12
	26435	Bo	G	03	J	01		23546	Nw	E	02	D	10		54326	Lf	B	02	J	10
2160	23456	Ne	D	04	K	11		23456	Bm	F	06	I	09		23456	No	K	05	D	10
	63254	Cm	I	09	J	10		42563	Ad	L	10	F	08		26435	Ad	A	03	E	01
	54326	Ad	G	04	H	11		26435	Bm	C	03	K	11		63254	Bm	B	02	H	08
	42563	Cm	E	05	F	06		54326	Wk	J	02	B	12		42563	Wk	A	03	E	01
	35642	Bo	C	06	L	09		35642	Bm	G	07	A	01		54326	Bm	D	04	L	12
	26435	Lf	H	08	G	07		63254	Ad	J	02	B	12		35642	Ad	I	07	C	05
1440	45326	Nb	B	02	A	01		23456	Bv	I	08	F	07							
	23546	Ak	B	02	J	10		54326	Ip	K	11	H	08							
	65423	Ak	E	05	A	01		35642	Ne	I	09	F	06							
	34265	C1	C	03	D	04		26435	Ip	G	07	D	04							
	46352	Nb	E	05	J	10		42563	Wm	G	03	L	09							
	52634	C1	K	11	L	12		63254	Wm	C	06	H	12							

Next to each lead in each block are shown the relevant group codes which occur in the 5-6 sections of that lead, so in the 2160s there are 3 occurrences of each code and in the 1440 there are 2 occurrences of each code.

The blocks containing Cambridge-over and Carlisle-over methods will between them yield all 23 methods from those groups once lead splices and 2nds/6ths place variants are taken into account.

You will notice that there are no London-over blocks in the chosen 2160s and 1440s. However, what I did was to extract the 6 highlighted leads of Norwich-over over methods from the 1440 and the second 2160 which together form a 6-lead block of their own (shown below left), and then converted this block to its London-over equivalent (below right) which contains all the same group codes and will yield all 6 London-over methods once lead splices are taken into account. The remaining leads of Norwich-over methods will between them yield all 12 Norwich-over methods once lead splices and 2nds/6ths place variants are taken into account.

23456	Ne	D	04	K	11
54326	Ad	G	04	H	11
42563	Wm	G	03	L	09
63254	Bm	B	02	H	08
26435	Lf	H	08	G	07
35642	Ad	I	07	C	05

45326	Cu	D	04	K	11
56243	Li	G	04	H	11
43625	We	G	03	L	09
64532	Ke	B	02	H	08
25364	Lo	H	08	G	07
32456	Li	I	07	C	05

So I now had a set of 48 leads which contained all 41 methods. However, all the Cambridge-over and Norwich-over leads were in-course and all the Carlisle-over and London-over leads were out-of-course which wasn't good news for obtaining a composition with a change of backwork every lead; every in-course section of the composition would need to alternate between Cambridge-over and Norwich-over methods, and every out-of-course section of the composition would need to alternate between Carlisle-over and London-over methods; it seemed likely that such a composition would be difficult to produce, or that it would require many singles to alternate between in-course and out-of-course sections to obtain a change of backwork every lead.

So what I did was transpose the first 2160 so that course 54236 became the plain course (which meant every other course transposed accordingly), this changed the Cambridge-over and Norwich-over blocks of the 2160 to be out-of-course, and the Carlisle-over block to be in-course.

54236	Du
25364	Bv
64532	Yo
43625	Bv
32456	Bv
56243	Bo

23456	Ch
35642	Nw
26435	Sa
63254	Ch
42563	Sa
54326	Nw

54236	Ne
43625	Wm
25364	Lf
56243	Lf
64532	Lf
32456	Lf

So that meant I now had 4 in-course blocks, being 1 Cambridge-over, 1 Carlisle-over and 2 Norwich-over; and 4 out-of-course blocks, being 1 each of Cambridge-over, Norwich-over, Carlisle-over and London-over. This meant the prospect of arranging the composition to have a change of backwork every lead was now far greater, although in the in-course sections every alternate lead would need to be Norwich-over. Ideally one of the in-course Norwich-over blocks would have been London-over, but I was unable to find any combinations of blocks which would achieve this.

The final piece of the jigsaw was to arrange the leads into a composition which included all 41 methods and had a change of backwork every lead. All my previous compositions of Minor had contained two particular features, firstly a plain lead of every method and secondly no repetitions of lead-heads or lead-ends between calls within any one block. I wanted to make sure the second feature was retained, but this meant the first could not be (a minimum of 10 calls would be required per part), however I made sure that all the methods which did not have plain leads were all ones which did not have both 2nds and 6ths place variants (i.e. methods with 2nds or 6ths place bell as pivot, or London-over methods), thus they were still uniquely defined even though there was a call every lead. I was able to produce such a composition with a change of backwork every lead, but then also decided I would add the feature of a change of frontwork every lead too, this did make things more challenging but eventually I was successful in producing a composition. This can be found on the following page and was rung at Birmingham Cathedral on 28th August 2012.

Having completed the initial challenge, I then went on to generate some normal length compositions containing 39 methods (with 3 options on which methods were missing), and some additional 5760s in 41 methods which don't have a change of backwork every lead so would obviously be easier to ring.

5760 Spliced Surprise Minor (41 methods all-the-work)

Containing a change of backwork and frontwork every lead

23456 Bm
42635 Ch
- 42356 Ab
56234 Sa
25463 Bm
- 34256 Wo
62534 Wk
- 56234 Cm
34625 Ws
- 34256 Ip
23645 Wm
45362 Mu
- 45623 Bc
36245 Nf
52436 Bc
- 64523 Bk
42635 No
- 64235 Bo
52364 Ad
s 25643 Nb
43562 Cu
s 34625 Pr
42356 Mo
56234 Wk
- 25634 Nw
46325 Ws
s 64253 Ak
32564 He
26345 Lf
- 64532 We
- 25643 Ne
36425 Du
- 36254 Cl
- 36542 Ro
64325 Wh
42653 Ro
- 25364 Cl
56243 Li
43625 St
32456 Ke
- 56432 Bv
32645 Lf
45263 Su
63524 Lo
- 45632 Hu
32564 Ct
64253 Co
53426 Yo
s 35264
Repeat 4x

Method Abbreviations:

Allendale (Ad), Alnwick (Ak), Annable's London (Ab), Bacup (Bc), Bamborough (Bm), Berwick (Bk), Beverley (Bv), Bourne (Bo), Cambridge (Cm), Canterbury (Ct), Carlisle (Cl), Chester (Ch), Coldstream (Co), Cunecastre (Cu), Durham (Du), Hexham (He), Hull (Hu), Ipswich (Ip), Kelso (Ke), Lightfoot (Lf), Lincoln (Li), London (Lo), Morpeth (Mo), Munden (Mu), Netherseale (Ne), Newcastle (Nw), Norfolk (Nf), Northumberland (Nb), Norwich (No), Primrose (Pr), Rossendale (Ro), Sandiacre (Sa), Stamford (St), Surfleet (Su), Warkworth (Wk), Wearmouth (Wm), Wells (We), Westminster (Ws), Whitley (Wh), Wooler (Wo), York (Yo).

I thought it would be of interest to try and “un-pick” some of the complementary blocks to try to get some idea about why they work.

Looking first at the 1440 used for the change-of-backwork-every-lead composition, if you change the lead of Bv to Du (course splice) and then change all the leads of Du to Yo and all the leads of Wm to Lf (5 3-lead splices), then all the methods have a place notation of 14 between the negative rows in the 5-6 sections.

Looking at the first 2160, if the 5-6 sections of the leads of Wm are swapped with the 5-6 sections of the lead of York, these leads become Lf and Du; if the lead of Ne is then changed to Lf (course splice) then the Norwich-block consists of 6 leads of Lf so these can be discounted. All the remaining methods have a place notation of 34 between the negative rows in the 5-6 sections.

The second 2160 is a bit more difficult to interpret, the way I broke it down was as follows. There are 5 different leads of Ad, the missing lead 23456 means you are missing groups D, K, 06 and 09. There are 5 different leads of Bm, the missing lead 42563 means you are missing groups E, J, 05 and 10. There are 4 remaining leads which have the place notation 14 between the negative rows in the 5-6 sections, once grouped together the missing groups from a full set of 12 are A, B, C, L, 01, 02, 03 and 12. The remaining leads of Bo, No and Wk provide you with all the missing groups, i.e. A, B, C, D, E, J, K, L, 01, 02, 03, 05, 06, 09, 10 and 12.

John S Warboys
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